Bayes Filter

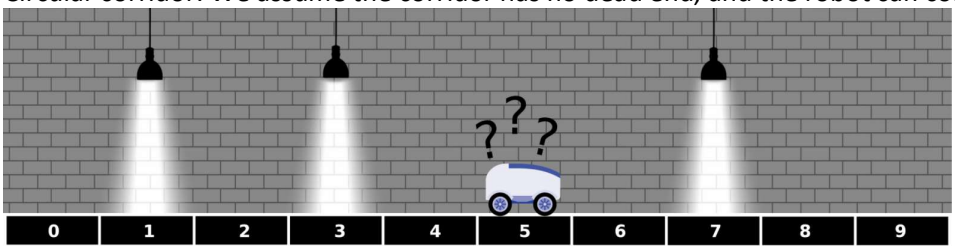
Saturday, January 28, 2023 2:48 PM

Applied to a process a filter takes noise (uncertainty) out of an input to get a more accurate estimate as output. The term Bayes is applied because concepts from Bayes statistics are involved. A Bayes Filter is an estimation algorithm that uses past values to provide a more accurate estimate.

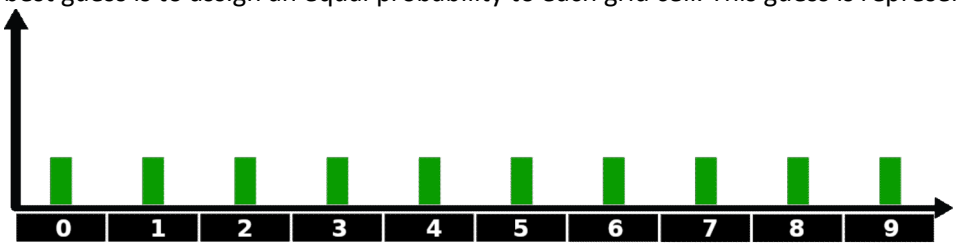
Bayes Filter Localization Example:
In this example let's assume a long corridor as a 1 dimensional robot localization, whose only features are three spotlights pointing downwards from the ceiling. Using its light sensor the robot can identify if it's standing below a spotlight or standing in dark area. The robot cannot identify in which spotlight is located , since all three spotlights are equal.

Now imagine that someone else puts the robot somewhere in the corridor, and we don't know where. We do have a map of where the corridor spotlights are located, and we can read the output of robot's light sensor. Additionally we can command the robot to move forward or backward.

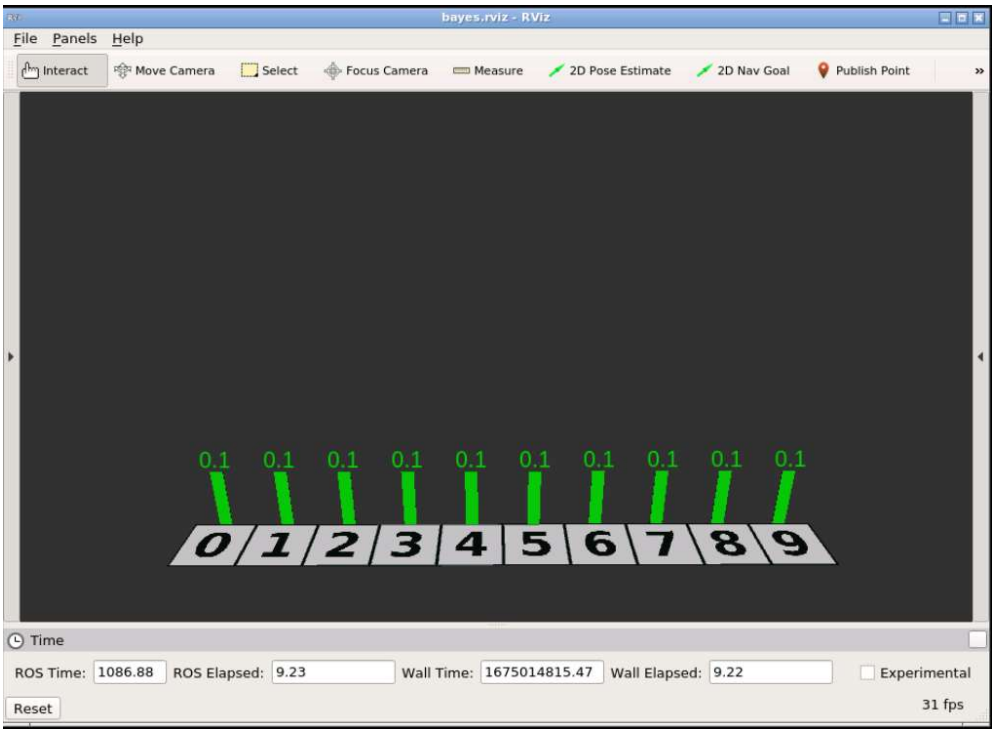
Simply assumptions
Discretization of space: We assume the corridor is a grid of 10 different positions. The robot can only move in discrete steps.
Circular corridor: We assume the corridor has no dead end, and the robot can continue moving forward indefinitely.



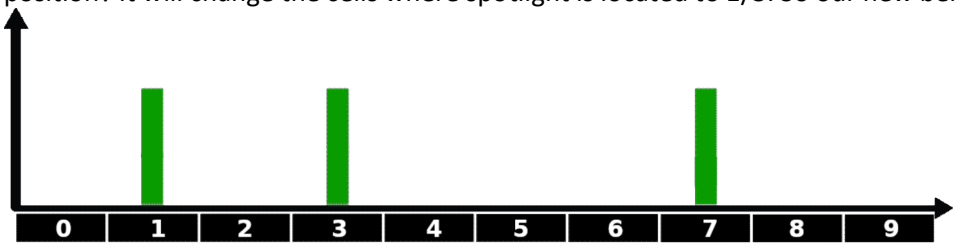
Bayes Filter Initialization
In the beginning we have no absolutely information about the robot's position. Since all grid are equally possible, our best guess is to assign an equal probability to each grid cell. This guess is represented in the below image.



If the corridor is discretized in 10 grid cells, what is the probability that the robot is in any given cell?
0.1



Now let's imagine that our robot's light sensor has detected a light. How does it change our belief about the robot position? It will change the cells where spotlight is located to 1/3. So our new belief is:



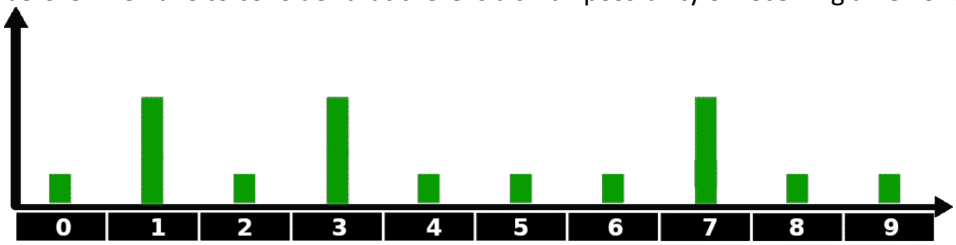
This logic is correct but we are still missing one important factor: sensor noise

Sensor Noise

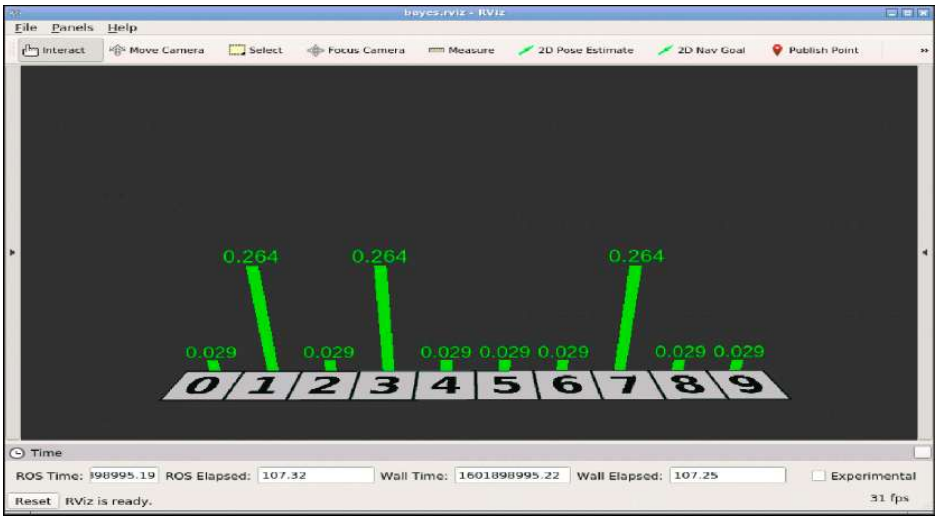
To be able to perceive, robots must rely on sensors, but because sensors have several imperfections, a key challenge that any robot system has to overcome is dealing with those limitations.

Likelihood: Let's consider an imperfect light sensor that is subject to noise. Given that the robot's sensor has detected a light, how would our belief changes?

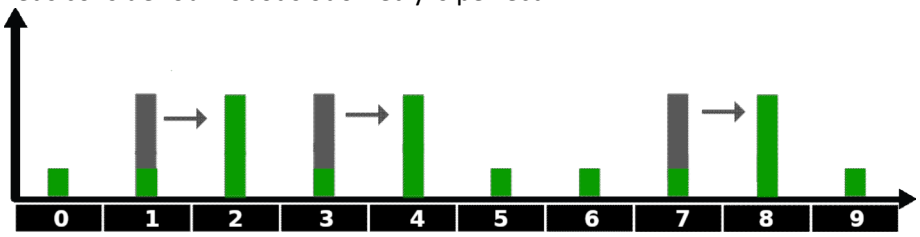
Because our light sensor is noisy, we cannot attribute 1/3 of the probability to each illuminated grid cell as we did before. We have to consider that there is a small possibility of receiving an error sensor measurement.



As we can see the sum of all the probabilities is greater than one, so we need to normalize it.



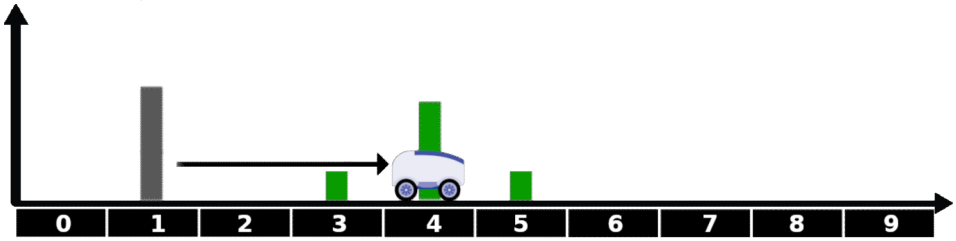
Now let's incorporate into our analysis the fact that the robot can move along the corridor. For that purpose, let's move the robot one grid cell per step to the right side. How does it change our belief?
Let's consider our robot's odometry is perfect.



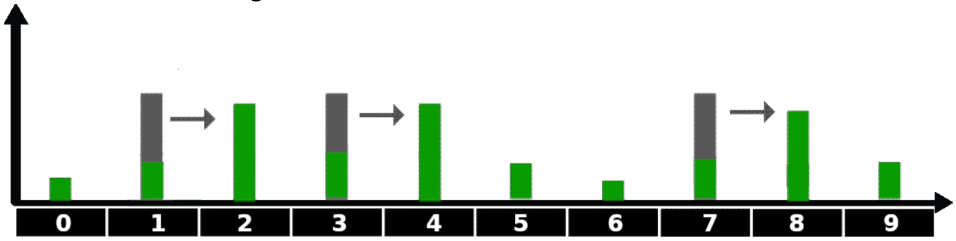
Each peak indicates a grid cell where the robot is very likely to be currently. If we command the robot to move one grid cell to the right, our belief of where the robot currently is also shifts one grid cell to the right.

Movement Noise

With noise sensor, odometry data will not report the robot's real position in a perfect manner.
In that case, the robot's real movement can be somewhat less or somewhat more than the odometry indicates.



This image shows a robot that is originally placed in cell number one. The it receives a command to move tree cells forward. We represent the robot's estimated position after the movement by assigning a low probability, for instance 10% for shorter movement, a high probabilty value, for instance 80%, to a perfect movement, and again a low probabilty value of 10% to a longer movement.



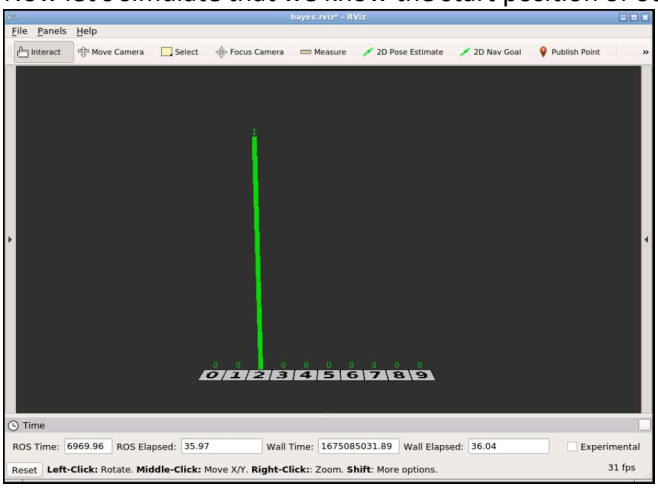
Predict Step as a Sum of two Probabilty Density Functions
To model a robot movement, we have to move a belief (PDF) by the same distance and combine it with the PDF that characterizes the movement noise.

Sum of PDF's:

$f_{U+V}(X) = f(U * V)(x)$

A simple description of convolution reads as follow: We model one PDF as a sliding window and then move it over the other one. The PDF that slides is called the Kernel. In the context of the Discrete Bayes Filter, a convolution is a mathematical operation in which the Predict Step with noisy movement is calculated.

Now let's simulate that we know the start position of our robot, in this case our belief is:

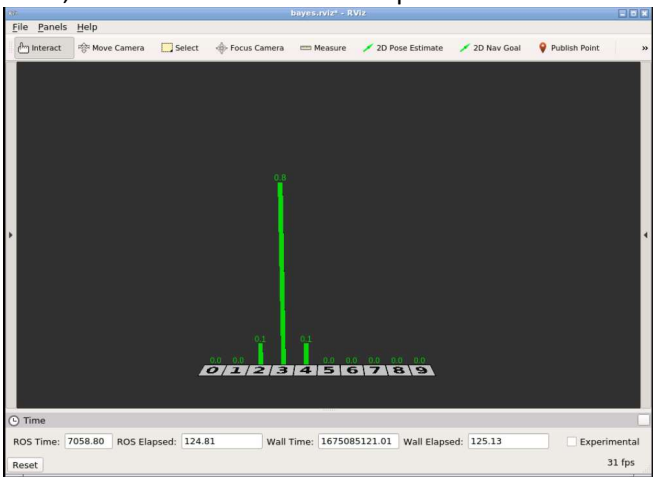


Shift or move the belief one step :

$\mathrm{F}_{U+V}(x) = [0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] * [0.1 \quad 0.8 \quad 0.1]$

$F_{U+V}(x) = [0 \quad 0 \quad 0.1 \quad 0.8 \quad 0.1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$

Then, we move the robot one step.



Shift or move the belief one step :

$F_{U+V}(x) = [0 \quad 0 \quad 0 \quad 0.1 \quad 0.8 \quad 0.1 \quad 0.0 \quad 0 \quad 0 \quad 0] * [0.1 \quad 0.8 \quad 0.1]$

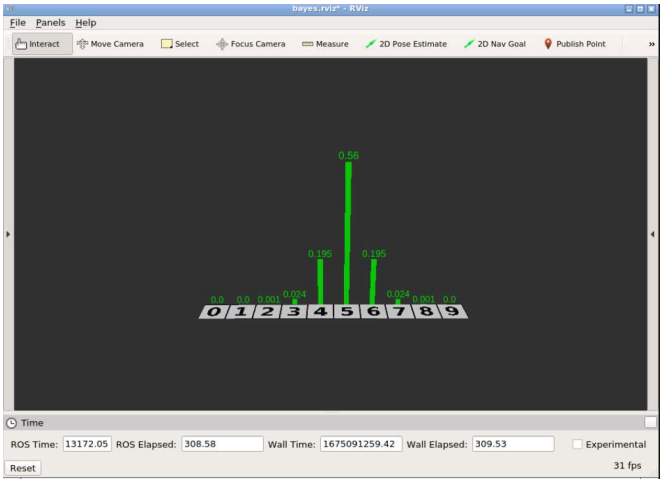
$F_{U+V}(x) = [0 \quad 0 \quad 0.01 \quad 0.16 \quad 0.66 \quad 0.16 \quad 0.01 \quad 0 \quad 0 \quad 0]$

Execute Next step:

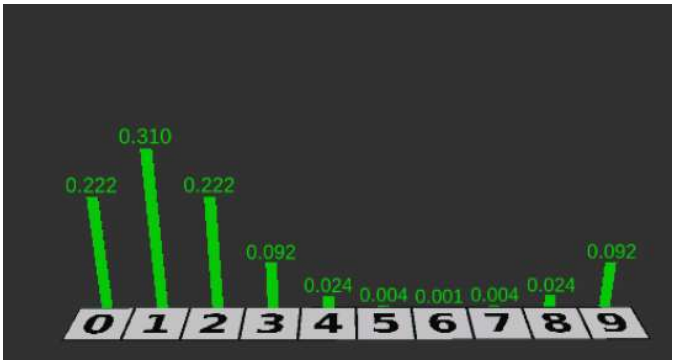
Shift or move the belief one step :

$F_{U+V}(x) = [0 \quad 0 \quad 0 \quad 0.01 \quad 0.09 \quad 0.66 \quad 0.09 \quad 0.01 \quad 0 \quad 0] * [0.1 \quad 0.8 \quad 0.1]$

$F_{U+V}(x) = [0 \quad 0 \quad 0.001 \quad 0.024 \quad 0.195 \quad 0.56 \quad 0.195 \quad 0.024 \quad 0.001 \quad 0]$



In each movement our robot earns uncertain. So we can say that the predict step decrease our estimated state accuracy



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Real strength comes from repetition

We have now finished the theory behind the discrete Bayes Filter and you might be wondering why the robot's location estimate is not accurate, or just ambiguous. We got probability values that are not very high and we got a belief vector with three peaks, indicating three possible robot locations (those grid cells with the highest green bars). Why?? Well, the algorithm did not produce a precise estimation because we executed just one filter cycle, which is not enough to provide a precise estimation. **The real power of the Bayes Filter comes from repeatedly applying the predict and correct steps in sequence, many, many times.**